Decay Scheme of Rb^{83}†

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We have re-examined the decay of Rb^{83}, which is reported to decay by electron capture, with emission of a single γ ray of 525 keV, to the 1.86-h isomer of Kr^{83}. With the aid of a lithium-drifted germanium detector and with a high-resolution beta spectrometer we find that the 525-keV photopeak observed with NaI crystals is really a superposition of three γ rays at 521 keV (46%), 530 keV (31%), and 553 keV (16%). An additional γ ray was found at 790 keV with intensity of 0.9%. Various coincidence experiments showed that only the 521- and 530-keV transitions lead to the 1.86-h Kr^{83m}. The 553- and 790-keV transitions lead to the 9.3-keV level of Kr^{83}. Direct electron capture to this level has also been observed with an intensity of 6%. The fraction of Rb^{83} disintegrations bypassing the isomeric state of Kr^{83} is 23%. The following new levels were assigned to Kr^{83}: 562, 571, and 799 keV. The total disintegration energy of Rb^{83} was shown to be $\geq$825 keV.

THE decay of Rb^{83} has been studied by a number of investigators.[1–3] This isotope decays entirely by electron capture with a half-life of 83 days. It is reported[3] to emit x rays, only one γ ray of 525±7 keV, and to decay entirely to the 1.86-h metastable state of Kr^{83}. The decay scheme of the latter isomer has been studied extensively.[4–7]

Measurements of Rb^{83} formation cross sections carried out in recent years[8–10] in this laboratory gave discrepant results when different counting techniques were used. Specifically, cross sections obtained when the Rb^{83} activity was determined by counting[8,9] the 525-keV γ ray were 15-20% higher than those obtained by gas counting of the Kr^{83m} daughter.[10] Direct comparison of the rates of disintegration of Rb^{83} sources deduced from the two different methods confirmed that there is indeed a real discrepancy and that the published decay scheme is inadequate. In particular it appeared that part of the decay of Rb^{83} bypasses the Kr^{83m} state. The decay scheme of Rb^{83} was therefore re-examined.

PREPARATION OF SAMPLES

Radiochemically pure Rb^{83} was prepared from purified 32.4-h Sr^{83} samples. The latter was isolated as a spallation product from silver targets bombarded with 3- or 30-GeV protons. The purified Sr^{83} in solution was allowed to decay for at least one week and the strontium then precipitated as nitrate. The solution was scavenged with iron precipitated as hydroxide. Carrier-free samples of Rb^{83} were obtained by evaporation of the purified solutions. The Rb^{83} prepared in this way is completely free from other rubidium activities.

More intense sources were made by bombarding ammonium bromide with 40-MeV α particles degraded by about 80 mg/cm^2 of aluminum. Under these conditions a small proportion of Rb^{84} is formed. In samples which were a few months old the Rb^{84} contamination was only a few per cent of the total activity. Final purification was by means of a Dowex-50 cation exchange column.

COMPARISON OF Rb^{83} GAMMA DISINTEGRATION RATE WITH Kr^{83m} PRODUCTION

Pure preparations of Rb^{83} (obtained from Sr^{83}) were used in these experiments. The absolute intensity of the photopeak at 525 keV was determined with a standard calibrated 3-×3-in. NaI crystal. Two methods were used to separate the Kr^{83m} quantitatively. In the first, the Rb^{83} activity was dissolved in water and transferred quantitatively into a glass bulb containing a measured amount of krypton carrier. A day later the krypton was swept out of the bulb and the solution by a stream of helium. The krypton was isolated and purified by gas chromatography. Quantitative recovery of the gas was always obtained. The purified krypton was transferred into a cylindrical counter (2 cm in diameter and 30 cm long) and P-10 counting gas was added to atmospheric pressure. The counters were operated in the proportional region. In a separate series of experiments it was shown that all disintegrations occurring within the active volume were counted. In the second method of separating Kr^{83m} from Rb^{83}, a sample of the latter was mounted on a platinum ribbon filament and placed in a special vessel containing a measured amount (about 1 ml STP) of krypton carrier. A day later the filament was heated to bright redness until all of the rubidium had sublimed. The krypton was then isolated, purified, and counted as in the first method. The results of these experiments are summarized in Table I. It is seen that the 525-keV γ disintegration rate of Rb^{83} is 21% higher

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* On leave of absence from the Weizmann Institute of Science, Rehovoth, Israel.
[1] D. G. Karraker and D. H. Templeton, Phys. Rev. **80**, 646 (1950).
[2] S. V. Castner and D. H. Templeton, Phys. Rev. **88**, 1126 (1952).
[3] M. L. Perlman and J. P. Welker, Phys. Rev. **100**, 81 (1955).
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[5] I. Bergström, Phys. Rev. **81**, 638 (1951); Arkiv Fysik **5**, 191 (1952).
[6] P. T. Barret, Proc. Phys. Soc. (London) **65A**, 450 (1952).
[7] S. L. Ruby, Y. Hazoni, and M. Pasternak, Phys. Rev. **129**, 826 (1963).
[8] H. R. Fickel and S. Katcoff (private communication).
[9] S. Katcoff and A. Wyttenbach (private communication).
[10] I. Dostrovsky and R. W. Stoenner (private communication).

than that of the Kr^{83m} in equilibrium with it. The possibility that the observed excess intensity in the 525-keV photopeak might be due to annihilation radiation (511 keV) was investigated. Careful 511–511-keV coincidence measurements failed to show any significant positron emission. There remains the possibility that the cause of the discrepancy is another γ transition close to 525 keV which bypasses the isomeric level in Kr^{83} at 41 keV. Therefore the γ-ray spectrum was very thoroughly investigated.

GAMMA-RAY SPECTRUM

The electromagnetic radiation spectrum in the energy range 3 keV to 1 MeV was re-examined. With a 10-cm-diam argon-filled proportional counter the only radiations observed in the energy range 3–100 keV were those at 9.3 and 12.6 keV. The 12.6-keV radiation is the K x ray of Kr, and the 9.3-keV transition has been reported[5,6] for Kr^{83m}. The latter was observed, at raduced intensity, even in freshly prepared samples of Rb^{83} free of Kr^{83m}. With NaI scintillation detectors only a large photopeak at 525 keV and a weak peak at 790 keV were observed in addition to the broad peak at 12 keV. The radiations were then investigated with detectors of greater resolution.

A spectrum taken with a lithium-drifted germanium detector 2.4 mm thick and 2.7 cm^2 in area showed photopeaks at 521, 531, and 553 keV in place of the single photopeak at 525 keV observed with NaI crystals. The corrected relative intensities of the three peaks are 100, 66, and 36, respectively (see Fig. 1). The ratio of the total γ intensity to the sum of the 521- and 531-keV transitions, namely, $202/166=1.22$, is remarkably close to the ratio, 1.21, of "525"-keV γ disintegrations to Kr^{83m} disintegrations shown in Table I. This strongly suggests that the 521- and 531-keV transitions populate the 41-keV isomeric level in Kr^{83} while the 553-keV transition bypasses it. Evidence for this is provided by the coincidence experiments described below.

A double-focusing $\pi\sqrt{2}\beta$ spectrometer was used to investigate internal conversion electrons in the energy range 15 to 540 keV. Some experiments were also carried out with an external converter of uranium foil (18 mg/cm^2). In both experiments electrons were observed which corresponded to transitions at 521.0 ± 0.5 keV and 530.0 ± 0.7 keV. A careful search was also made in the low-energy region; no conversion electrons were found other than those already reported for Kr^{83m}. The source used in these experiments was of insufficient intensity for the detection of minor transitions. We did not observe K/L ratios nor the electrons corresponding to the 553-keV transition. For the same reason the accuracy of the γ-intensity ratios, and of the conversion electron intensity ratios was rather poor. The K-conversion electron intensity from the 521-keV transition is (3.0 ± 0.5) times that of the 530-keV transition. This corresponds to a ratio of 2.0 ± 0.5 for the K-conversion coefficient of these transitions. An estimate of the average K-conversion coefficient of the three transitions could be made from the measurements with a Gerholm spectrometer (see below). The value obtained is $1\text{–}2\times10^{-3}$, and is consistent with $E1$, $M1$, or $E2$ transitions. The theoretical K-conversion coefficients for these transitions are given by Rose[11] as 0.82, 1.83, and 2.64×10^{-3}, respectively.

Conversion electrons were also studied with a lithium-drifted silicon detector (resolution 20 keV). The three electron groups could not be completely resolved with our detector, the higher energy line appearing as a shoulder in the electron spectrum. After a suitable

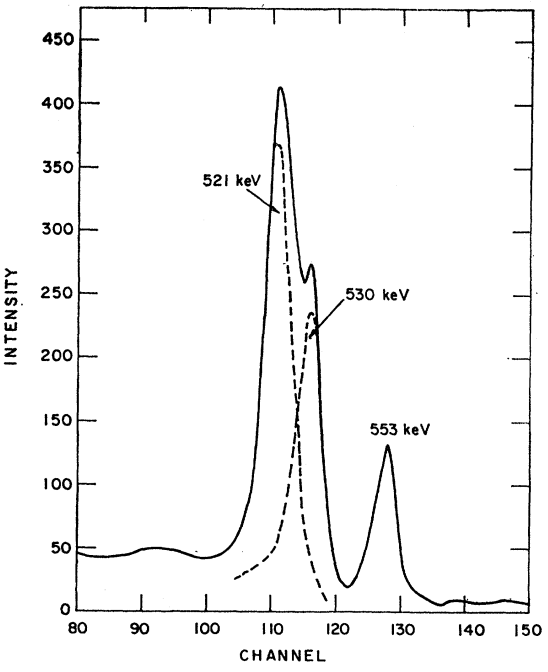

FIG. 1. Gamma-ray spectrum of Rb^{83} in the vicinity of 525 keV taken with a lithium-drifted germanium detector 2.4 mm thick and 2.7 cm^2 area. Full width at half-maximum: 8 keV.

TABLE I. Comparison of γ counting of Rb^{83} with gas counting of Kr^{83m} daughter in equilibrium.

Exp. No.	Method of Kr^{83m} separation	Absolute 525-keV photopeak intensity γ/min.	Kr^{83m} disintegration rate dis/min	Ratio (γ/gas)
1	solution sweeping	199 400	105 000	1.208
2	solution sweeping	176 800	142 600	1.239
3	solution sweeping	19 060	15 710	1.213
4	Rb sublimation	31 270	26 230	1.192
5	Rb sublimation	29 520	24 740	1.193
Mean				1.209

[11] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

unfolding procedure (using the line shape of the conversion electrons from Cs^{137}) the intensity of the K-conversion electrons of the 553-keV transition was estimated. From the relative intensities of the various conversion electrons it is concluded that the K-conversion coefficient of the 553-keV transition is the same as that of the 530-keV transition (within a factor of 2).

To gain further insight into the nature of the various γ transitions, a series of coincidence experiments was carried out.

CONVERSION ELECTRON-γ COINCIDENCES

A Gerholm lens spectrometer was used which was modified by Takahashi, McKeown, and Scharff-Goldhaber[12] to pre-accelerate the electrons emitted from the source. We employed 20 kV of pre-acceleration which permitted detection of the low-energy electrons by an anthracene scintillator. The γ rays from the source were detected by a 1-$\times 1\frac{1}{2}$-in. NaI crystal. The signals from the two detectors, after appropriate amplification, were fed to a coincidence circuit (resolving time $2\tau = 100$ nsec) and a multichannel analyzer.

The conversion electron spectrum was usually measured as a function of time after separation of the Kr^{83m}. The lower half of Fig. 2 shows a typical spectrum after some of the 1.86-h Kr^{83m} had grown in. At zero time the K and L lines belonging to Kr^{83m} are absent while the conversion electrons from the 9.3-keV transition are present, though in considerably lower intensity relative to the Auger electrons. In these coincidence experiments use was made of the fact that the 9.3-keV level has a lifetime of 147 nsec.[7] Therefore the observation of delayed coincidences uniquely identifies transitions to this level. In one set of experiments the electron spectra coincident with "525"-keV γ rays were recorded both with and without a delay of 180 nsec of the γ-ray pulse (top half of Fig. 2). The delayed coincidence spectrum shows clearly that there is a transition to the 9.3-keV level involving at least one of the γ rays near 525 keV. With no delay of the γ-pulse Auger electrons are also recorded in the coincidence spectrum.

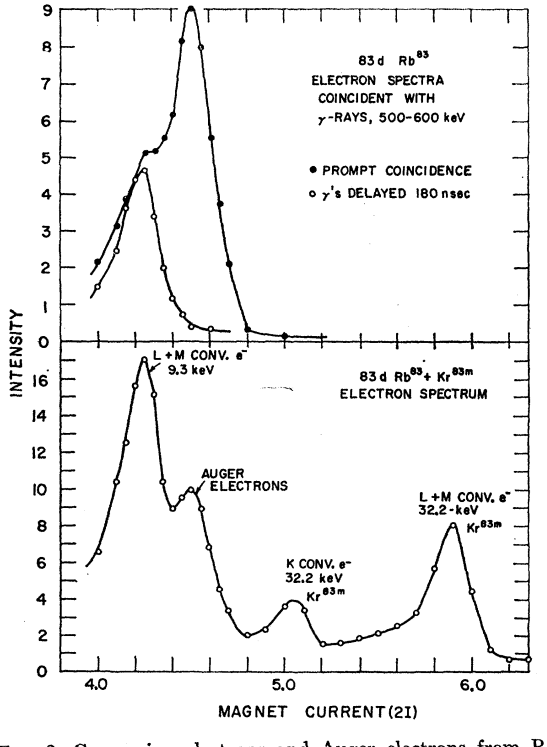

FIG. 2. Conversion electrons and Auger electrons from Rb^{83} observed with a Gerholm lens beta-ray spectrometer, 20-kV pre-acceleration. Lower half: "Singles" spectrum taken after some Kr^{83m} had grown in. Upper half: Electron spectra taken in prompt and delayed coincidence with γ rays near 525 keV.

In another series of experiments γ spectra were observed in coincidence with selected portions of the electron spectrum. When the gate was set on the Auger electrons the γ-ray spectrum in prompt coincidence (Fig. 3, solid curve) was identical with the "singles" spectrum. The peak at 880 keV is due to a small amount of Rb^{84} present in this source and was useful for energy calibration. The γ-ray spectrum in delayed coincidence with 7.3-keV electrons (L-conversion electrons from the 9.3-keV transition) is shown in Fig. 3 (dashed curve). It is seen that the position of the main photopeak is now at 550 keV rather than at 525 keV. The 790-keV peak is at the same position in both spectra. From these experiments we conclude that the 553-keV transition is in coincidence with the 9.3-keV γ ray. In

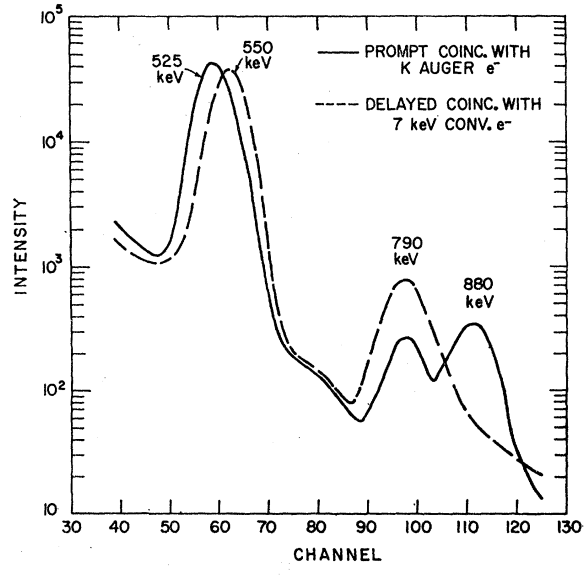

FIG. 3. The solid curve shows the γ-ray spectrum of Rb^{83} in prompt coincidence with K Auger electrons. The dashed curve shows the γ rays in delayed coincidence with conversion electrons from the 9.3-keV transition. The peak at 880 keV is from Rb^{84} present as a small impurity. Detector: 1- $\times 1\frac{1}{2}$-in. NaI crystal.

[12] K. Takahashi, M. McKeown, and G. Scharff-Goldhaber, Phys. Rev. **136**, B18 (1964).

the delayed coincidences with 7.3-keV electrons we measure only the 553- and 790-keV γ rays whereas in the prompt coincidences with Auger electrons we measure the total γ-ray spectrum. Therefore the relative intensity of the 550-keV photopeak to that of the 525-keV photopeak (normalized to the intensity of the 790-keV peak) observed in the two coincidence experiments represents the fraction of the total γ-ray intensity near 525 keV due to the 553-keV transition. The observed value is approximately 0.25. The corresponding ratio derived from the measurements with the lithium-drifted germanium detector is $36/202=0.173$.

X-RAY-γ AND γ-γ COINCIDENCES

Coincidence measurements were carried out between the signals from a 10-cm-diam argon-filled proportional counter and those from a 2-$\times$2-in. NaI crystal. The proportional counter detected the 9.3-keV γ ray and the 12.6-keV x-ray. The NaI crystal detected the γ-ray group at 525 keV. The signals from the latter detector were delayed and lengthened sufficiently to insure the recording of all coincident pulses from the slower proportional counter. The coincidence pulses were used to gate a two-dimensional analyzer operated in the 64$\times$64 channel mode. Analysis of the data thus obtained showed coincidences between the 9.3-keV x ray and the high-energy end of the 525-keV photopeak. Essentially no coincidences were observed with the low-energy edge of this photopeak. These results lend further support to the conclusion that the 553-keV transition feeds the 9.3-keV level.

The same arrangement was used to determine the ratio of the x-ray disintegration rate to the total "525"-keV γ-ray intensity (see below).

790-keV γ-RAY AND TOTAL DISINTEGRATION ENERGY

A γ ray of low intensity at 790 ± 5 keV was observed in all samples containing Rb^{83}, including those which were purified by cation exchange on a Dowex-50 column and those obtained from the decay of purified 32.4-h Sr^{83}. Measurements made with a NaI detector over a period of four months showed that this γ ray decays with the same half-life as the group of γ rays near 525-keV, and that its absolute intensity is $(1.0\pm0.1)\%$ of the combined intensity of the latter.

Coincidence experiments with the Gerholm lens spectrometer showed that the 790-keV transition feeds the 147-nsec 9.3-keV level (see Fig. 3). Thus a level in Kr^{83} at 799-keV is established. Since the total disintegration energy of Rb^{83} has been estimated as[13,14] $\sim$760 keV, it seemed possible that the decay of the 799-keV level proceeds largely by L-electron capture. In that case the intensity (relative to the 525-keV photopeak) of the 790-keV γ ray coincident with K Auger electrons ($\approx$11 keV) would be substantially reduced, or may even be zero. When this coincidence measurement was performed, no decrease in intensity (within the experimental uncertainty of 30%) of the 790-keV peak was observed relative to the photopeaks near 525 keV. It follows that L capture cannot account for more than about 30% of the total electron capture transitions, and therefore the decay energy to the 799-keV level is at least 25 keV. The total disintegration energy of Rb^{83} is thus $\geq$825 keV.

DIRECT DECAY TO 9.3-keV LEVEL

The fraction of Rb^{83} decaying directly to the 9.3-keV level (and/or ground state) of Kr^{83} was measured by two methods. In the first, the total absolute intensity of the three γ rays near 525 keV was compared with the absolute intensity of the 12.6-keV K x rays from Rb^{83} decay. In the second method,[3] the γ-X coincidence rate was compared with the K x-ray counting rate. The x-ray intensities were always measured very soon after removal of the 1.86-h Kr^{83m} daughter. Results from both experiments were in reasonable agreement and they showed that $(6\pm3)\%$ of the Rb^{83} decays directly to the 9-keV level (and/or possibly to the ground state). Details of each type of experiment follow.

In the first of the methods, four samples of Rb^{83} were mounted immediately after boiling off the 1.86-h Kr^{83m}. The absolute γ-ray intensity (sum of 521-, 530-, and 553-keV gammas) was determined with a calibrated 3-$\times$3-in. NaI crystal. The absolute Kr K x-ray intensity was measured with a beryllium-covered 2-mm-thick NaI crystal which was calibrated by means of a Y^{88} source. The ratio of fluorescence yields of the K x rays from the Y^{88} standard to those from the Rb^{83} was taken as 1.08. Since the thin NaI crystal did not resolve the 9.3-keV γ rays from the 12.6-keV x rays it was necessary to make a small correction (4.3%) for the contribution of this γ ray to the x-ray counting rate. A series of four measurements gave the following results for the ratio of the γ-ray intensity to total disintegrations: 0.938, 0.948, 0.918, and 0.978; mean value of 0.945.

In the second of the methods, the arrangement used was that of the coincidence experiments described above. The 9.3-keV γ-ray peak was fairly well resolved from the 12.6-keV x-ray peak so that a channel could be set on the latter to make the contribution of the former almost negligible. In one experiment 94-mg/cm^2 aluminum absorber was added to further minimize any possible effect of the 9.3-keV γ ray. On the γ-ray side, the channel was set to accept the photopeaks at 521, 530, and 553 keV. The counting efficiency of these γ rays was determined by measuring these same sources with a calibrated NaI crystal. Small corrections were made for chance coincidences (1.8%), for γ rays count-

[13] M. Yamada and Z. Matumoto, J. Phys. Soc. Japan **16**, 1497 (1961).

[14] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.), NRC-59-1-77.

ing in the x-ray counter (0.6%), and for background (2.5% for γ's, 1.2% for x rays). From the corrected "singles" and coincidence counting rates, and the measured over-all efficiency for counting the γ rays, it is possible to calculate the absolute x-ray intensity. From the absolute γ-ray and x-ray intensities the fraction of disintegrations proceeding through the three γ rays near 525 keV was calculated. The results from two such experiments were 0.889 and 0.929 with a mean of 0.909.

By combining the results of the two types of experiment, and by making allowance for the 0.9% branch decaying via the 790-keV γ ray, we find that $(6.4\pm3.0)\%$ of the decays of Rb^{83} proceed by electron capture directly to the 9.3-keV level (and/or ground state) of Kr^{83}.

DISCUSSION

On the basis of the data presented in the previous sections we propose the decay scheme presented in Fig. 4. The 521- and 530-keV transitions populate the metastable state in Kr^{83} at 41 keV. Since no coincidences were observed between the two γ rays they are not in cascade and therefore two levels at 562 and 571 keV are assigned to Kr^{83}. The 553-keV γ ray is in coincidence with the 9.3-keV transition, thus also leading to a level at 562 keV.

The low K-conversion coefficients observed for all three γ rays indicate transitions of low multipole order. If we assume that the disintegration energy of Rb^{83} is 850 keV we can calculate $\log(ft)$ values for the decays to the various levels. They are summarized in Table II. On the basis of $\log(ft)$ values of the electron capture transitions and the low multipole order of the γ transitions, only the following spin and parity assignments are reasonable for both the 562- and 571-keV levels, namely: $\frac{3}{2}-$, $\frac{3}{2}+$, and $\frac{5}{2}-$. The assignment of $\frac{3}{2}-$ to the 571-keV level is consistent with the $\log(ft)$ value of the decay of Rb^{83} (6.1), with the $\log(ft)$ value of the decay from[15] Br^{83} (5.4) and with an $M1$ transition for the 530-keV γ ray. The transition to the 9-keV level would be $M2$ and therefore of an intensity too low to observe.

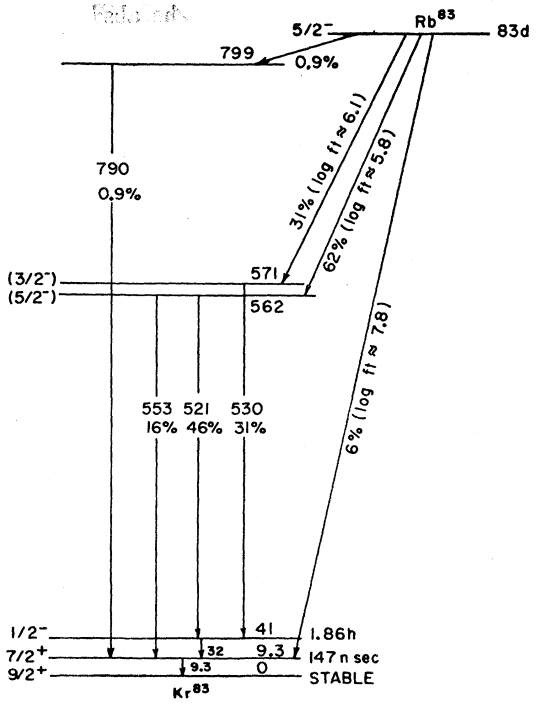

FIG. 4. Decay scheme proposed for 83-day Rb^{83}. Energy in keV.

Spin and parity assignments for the 562-keV level are less certain. The assignment of $\frac{5}{2}-$ to this level is consistent with the $\log(ft)$ for the decay of Rb^{83} (5.8) and with the fact that little if any Br^{83} decays through it.[15] It leads to the assignment of E2 for the 521-keV transition and is consistent with the somewhat higher observed K-conversion coefficient as compared with the $M1$ 530-keV transition. The assignment of $\frac{5}{2}-$ to the 562-keV level would require the 553-keV transition to the $\frac{7}{2}+$ 9-keV level to be of type $E1$. The relative intensities of the 553- and 521-keV γ rays indicate a retardation of the $E1$ transition which may be as high as 6×10^5. Such high retardation factors are not uncommon in $E1$ transitions between complex levels.

We have insufficient data to make spin and parity assignments to the 799-keV level.

In Fig. 4 we have indicated a direct electron capture process to the 9.3-keV level rather than to the ground state. This assignment is based on the experimentally observed intensity of the 9.3-keV γ ray in samples free of Kr^{83m} and on the observed $\log(ft)$ value of 7.8.

The decay scheme is consistent with the observation[16] that Br^{83} decays entirely through the 1.86-h Kr^{83m}.

ACKNOWLEDGMENTS

The present investigation was greatly facilitated by the generous cooperation of several colleagues who made their equipment available. We are grateful to the personnel at the Brookhaven Cosmotron and the 60-in.

TABLE II. $\log(ft)$ for decay of Rb^{83} to levels in Kr^{83} assuming total disintegration energy of 850 keV.

Level (keV)	Transition energy (keV)	$\log(ft)$
9	841	7.8
562	288	5.8
571	279	6.1
799	~50	~6.1

[15] M. Pasternak and T. Sonnino, Israel Atomic Energy Commission Report IA-827, 1963 (unpublished).
[16] T. Sonnino (private communication).

cyclotron for the irradiations; M. McKeown, K. Takahashi, and G. Scharff-Goldhaber for use of the lens spectrometer; M. L. Perlman for use of the $\pi\sqrt{2}$ double focusing spectrometer; A. Sunyar, C. Chasman, and R. A. Ristinen for taking the γ spectrum with their lithium-drifted germanium detector; and J. B. S. Waugh and R. C. Richardson for assistance in the measurement of conversion electrons with a lithium-drifted silicon detector. We are indebted to Dr. K. Takahashi and Dr. M. L. Perlman for valuable discussions, and to Dr. W. Rubinson for reviewing the manuscript.

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Influence of the Exclusion Principle on the Scattering Theory of Nonlocal, Separable Interactions*

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For a nonlocal, separable interaction which describes low-energy nucleon-nucleon scattering, the scattering-theory solution to the Brueckner-Bethe-Goldstone (BBG) equation is obtained analytically. The associated scattering amplitudes are evaluated for all momenta of the interacting pair. The off-energy-shell amplitudes are defined by analytic continuation to complex values of the relative momentum of the pair. The positions of singularities in the amplitudes are investigated for various values of both the two-body kinematical parameters and the density of the passive BBG Fermi gas. A comparison of the amplitudes calculated with and without incorporating the effects of the exclusion principle indicates that even for repulsive interactions the latter may differ from the former by as much as 50% for Fermi-gas densities as low as 1/500 of the observed nuclear density.

A. INTRODUCTION

THE purpose of this paper is the presentation of several results needed in the application of the theory of degenerate many-fermion systems[1] to the computation of the properties of nuclear matter. In particular we wish to treat nuclear matter as a low-density gas of fermions interacting via short-range forces.[2–4] For a perturbation-theory treatment of this system, one needs a solution of the independent-pair[5] two-fermion problem which incorporates the influence of both the exclusion principle and off-energy-shell propagation. In this paper we present such a solution for a case in which it can be obtained analytically.

The new result reported on herein consists of a solution to the Brueckner-Bethe-Goldstone (BBG) equation[6] with scattering theory boundary conditions which is valid for all values of the total momentum of the interacting pair and in which the initial relative momentum of the pair is treated as an arbitrary complex variable. We are able to obtain a solution to the BBG equation in closed form by utilizing a simple two-nucleon interaction, referred to as the WY interaction, introduced independently by Wheeler[7] and Yamaguchi.[8] Yamaguchi discussed the energy-shell scattering theory of this interaction for an isolated pair of nucleons. The energy-shell theory for arbitrary values of the total momentum of the pair was extended to low-density many-fermion systems by Verlet and Gavoret (VG).[9] Numerous more restrictive energy-shell calculations

* This work is based on a portion of a thesis submitted in partial fulfillment of the requirements for a Ph.D. degree at Princeton University, 1963.

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[1] See, e.g., J. Goldstone, Proc. Roy. Soc. (London) **A239**, 267 (1957): N. M. Hugenholtz, Physica **23**, 481 (1957); J. Hubbard, Proc. Roy. Soc. (London) **A240**, 539 (1957); V. Galitskii and A. B. Migdal, Zh. Eksperim. i Teor. Fiz. **34**, 139 (1958) [English transl.: Soviet Phys.—JETP **7**, 96 (1958)]; L. D. Landau, Zh. Eksperim. i Teor. Fiz. **30**, 1058 (1956); **32**, 58 (1957); **30**, 97 (1958) [English transl.: Soviet Phys.—JETP **3**, 920 (1957); **5**, 101 (1957); **8**, 70 (1959)]; D. F. Dubois, Ann. Phys. (N. Y.) **7**, 174 (1959); C. B. Duke, Ph.D. thesis, Princeton University, 1963 (unpublished); and A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, translated by R. A. Silverman (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963), Chap. 2.

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